Mrs G Gibson Mr D Keanan-Brown Mrs N Lee Mrs P Choong Mrs A Leslie



# **PYMBLE LADIES' COLLEGE**

# **YEAR 12**

# **MATHEMATICS EXTENSION I**

# **ASSESSMENT**

Time Allowed: 1 hour 30 minutes + 5 minutes reading time

Test date: 7th May, 2001

## **INSTRUCTIONS:**

- All questions should be attempted
- Start each question on a new page
- Approved calculators may be used
- A standard integral sheet is attached
- Write your name and your teacher's name on each page.
- DO NOT staple the questions together
- Hand this question paper in with your answers
- ALL rough working paper must be attached to the back of the last question
- A coloured sheet of paper must be attached, by stapling, to the end of each question
- This assessment task has a value of 15%
- There are 6 questions in this paper
- There are 9 pages in this paper

#### ASSESSMENT CRITERIA

- Provide answers which are complete, accurate and comprehensive
- Leave your answers in exact form unless otherwise stated
- Include all necessary working. Correct answers will not necessarily gain full marks unless necessary working is shown. Relevant working <u>might</u> gain marks even if your answer is wrong.
- Take care with mathematical notation
- Show relevant information clearly and unambiguously on sketches if required
- Present well set out solutions using a logical set of steps in which justification is included where necessary.

- (a) Find the acute angle between the lines y = -x and  $\sqrt{3}y = x$  (2 marks)
- (b) Find the indefinite integral of  $\int \frac{dx}{\sqrt{1-9x^2}}$  (1 mark)
- (c) If  $\alpha$ ,  $\beta$  and  $\lambda$  are the roots of the cubic equation  $2x^3 + x^2 x 2 = 0$  find the value of
  - (i)  $\alpha + \beta + \lambda$
  - (ii)  $\alpha\beta\lambda$
  - (iii)  $\alpha\beta + \alpha\lambda + \beta\lambda$
  - (iv) Hence, or otherwise, find the value of  $(\alpha 1)(\beta 1)(\lambda 1)$  (4 marks)
- (d) Given x = 12t and  $y = 6t^2$  write down
  - (i) the cartesian equation of the parabola
  - (ii) the coordinates of the focus
  - (iii) the equation of the directrix (3 marks)

(Start a new page)

(a) Solve the equation  $\sin \theta - \sqrt{3} \cos \theta = 1$  for  $0 \le \theta \le 2\pi$ 

(3 marks)

- (b) (i) Solve  $\sin 2x = \sin x$  for  $0 \le \theta \le \pi$ 
  - (ii) On the same number plane, sketch  $y = \sin 2x$  and  $y = \sin x$  for  $0 \le \theta \le \pi$  showing all important features.
  - (iii) Hence, or otherwise, find the area bounded by the curves  $y = \sin 2x$  and  $y = \sin x$  for  $0 \le \theta \le \frac{\pi}{3}$

(7 marks)

#### (Start a new page)

(a)

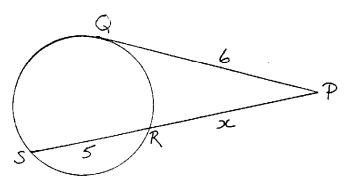
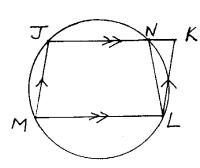


DIAGRAM NOT TO SCALE

PQ is a tangent to the circle QRS. PRS is a secant intersecting the circle in R and SGiven that PQ = 6, RS = 5 and PR = x, find x, giving reasons.

(3 marks)

(b)



DIAGRAM

NOT TO SCALE

The circle passes through the points J, N, L, and M. JKLM is a parallelogram Prove that NL = LK, giving reasons.

(3 marks)

(c)

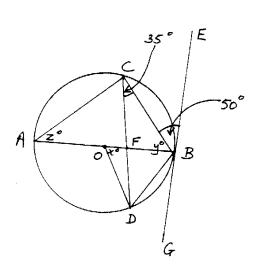


DIAGRAM NOT TO SCALE

0 is the centre of the circle. AB is a diameter. EBG is a tangent at B. If  $\angle CBE = 50^{\circ}$  and  $\angle DCB = 35^{\circ}$ , find the values of x, y and z, giving reasons.

(4 marks)

#### (Start a new page)

- (a) Find the general solution for the equation  $\tan \theta = \frac{1}{\sqrt{2}}$  (1 mark)
- (b) For the function  $f(x) = \sqrt{x} + 3$  find
  - (i) the inverse function
  - (ii) the domain of the inverse function.

(3 marks)

- (c) (i) Differentiate  $y = \tan^{-1} \frac{1}{x}$ ,  $x \neq 0$ 
  - (ii) Hence show that

$$\frac{d}{dx}\left[\tan^{-1}x + \tan^{-1}\frac{1}{x}\right] = 0$$

(iii) Then sketch the curve  $y = \tan^{-1} x + \tan^{-1} \frac{1}{x}$  (6 marks)

## (Start a new page)

- (a) The function  $f(x) = x^3 x^2 x 1$  has a zero between 1 and 2
  - (i) Taking x = 2 as a first approximation to this zero, use Newton's method to calculate a second approximation.
  - (ii) Would x = 1 have been a suitable first approximation to use? Explain your answer fully.

    (4 marks)
- (b) The function  $f(x) = ax^3 + bx^2 + cx + d$  has a double zero at x = 1 and a minimum value of -4 when x = -1.

Find the values of a, b, c and d.

(6 marks)

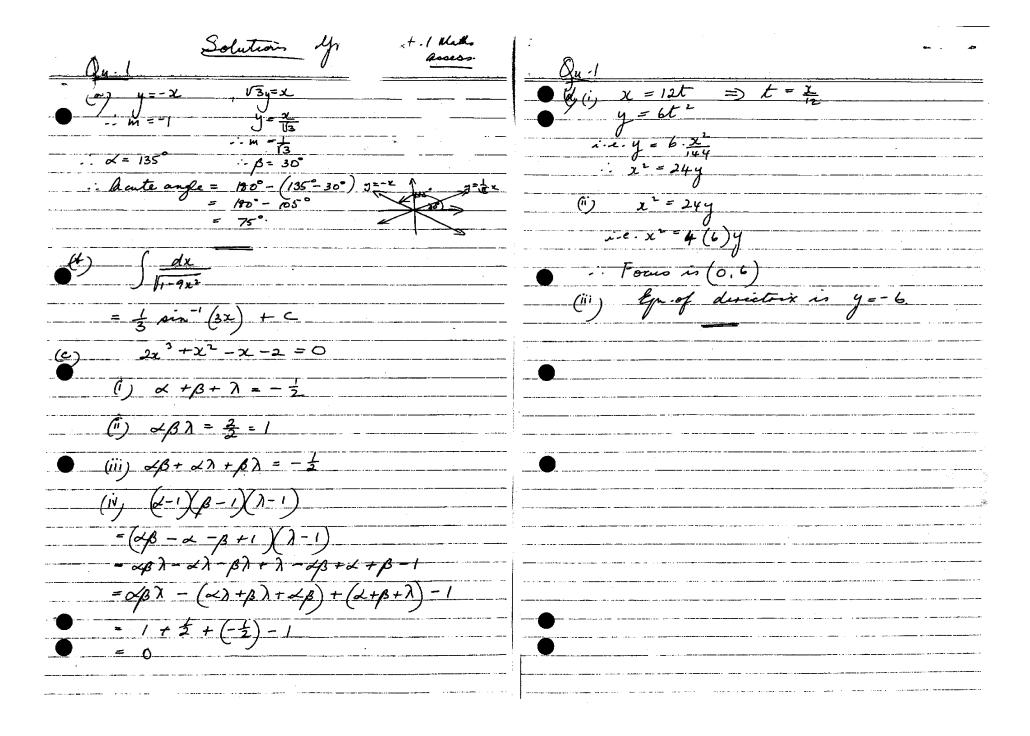
(Start a new page)

Consider the parabola  $x^2 = 4ay$ 

- (i) Find the equation of the tangent at  $P(2ap, ap^2)$
- (ii) If the tangent at P cuts the y-axis at T, show that  $T = (0, -ap^2)$
- (iii) Find the equation of the normal at P.
- (iv) If the normal at P cuts the y-axis at N, show that  $N = (0, 2a + ap^2)$
- (v) Explain why NT is the diameter of the circle passing through PTN. Hence find the equation of the circle.
- (vi) If the tangent at P cuts the x-axis at R and M is the midpoint of RN, show that R = (ap,0) and find the co-ordinates of M.
- (vii) Determine the equation of the locus of M and describe this locus geometrically.

(10 marks)

END OF PAPER



sin 8- 13 con 0 = 0=0=2T 2 sm/0-II tan0=13 -- 0 = 5 or 75 0 40 £ 21T pin x (2cox -1) -0 X=0, 5 or T. (ï) = \[ \frac{\cosx}{2} + \cosx \] \[ \frac{3}{3} \] = (-(+1) = + unit

(23 a) 
$$6 = x(5+x)$$
 (equare of tanget = product of intercepts)

 $x^2 + 5x - 36 = 0$ 
 $x = 4$  or  $-9$ 

but  $x > 0$  ( $x$  is a length)

 $x = 4$ 

b)  $x = 4$ 

b)  $x = 4$ 

b)  $x = 4$ 

c)  $x = 4$ 

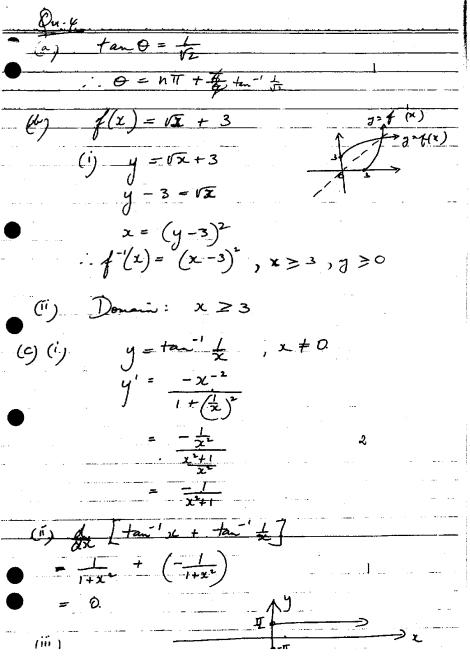
b)  $x = 3mL$  (ext.  $x = 1int$ . opp in cyclic quad)

 $x = 4$ 
 $x = 4$ 

c)  $x = 4$ 

Common errors: incomplete or missing reasons: false statements, especially in part (a).

= 70



Junie f'(1) =0 , f(1) ha

= ax3+ bx +cx+d = a + b + c + d = 0 2d=-4 d=-2 3(2-c)+c=06-3c+ c =0 6-20=0 2c = p a+3-2 =0

. Qu 6		K	1
(i)  y = -i		May	0
When x = 2	σ-, y=p-	0 /R	
p = y	•		
$px - 2ap^{2}$ $y = px$ $wlen$	- ap		
· · T = (	$x = 0$ $0 - ap^{2} = -ap^{2}$ $0 - ap^{2}$		
	normal at P	- xs :	
- x + 2ay	$\frac{y-ap}{2x-2ap}$ $= py-ap^{3}$		
(M) when	-2ap-ap3	= 0	
-y=	$= 2ap + ap^{3}$ $2a + ap^{2}$		

Ou b Ents) 10 900 LN PT - 90 ( angle between mg an angle is a semicircle is 90°.

NT is a diameter of the curile passing those PTN Centre of this civile = 2a + ap Ego of circle is X 1 px-ap サニストセス = a + 2x